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14. ABSTRACT
The Randomized Graph Coloring research aims to provide an efficient randomized algorithm for solving a specific NP-Hard problem, that of graph 3-coloring problem. The simulations are designed to test the practical behavior of the algorithm. Research in graphs and matroids will provide results publishable in top journals in Discrete Mathematics that generalize the existing literature. The objective of the connectivity research is to provide results about longest cycles and contractible edges in graphs and matroids. Such results have applications in combinatorial optimization problems.

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FINAL TECHNICAL REPORT

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OBJECTIVE: The Randomized Graph Coloring research aims to provide an efficient randomized algorithm for solving a specific NP-Hard problem, that of graph 3-coloring problem. The simulations are designed to test the practical behavior of the algorithm. Research in graphs and matroids will provide results publishable in top journals in Discrete Mathematics that generalize the existing literature. The objective of the connectivity research is to provide results about longest cycles and contractible edges in graphs and matroids. Such results have applications in combinatorial optimization problems.

APPROACH The Randomized Graph Coloring research was conducted using the Origin 2000 parallel processing IRIX supercomputer at The University of Mississippi. Large graphs were randomly generated using the Mathematical Software package MATLAB. The efficiency of the improved anti-voter coloring scheme was tested on these large graphs. The martingale approach to the convergence of the algorithm was also investigated. The research on longest cycles focused on generalizing results from 2-connected graphs and matroids to 3-connected graphs and matroids. We also worked on connectivity problems for graphs and matroids. Problems concerning covering contractible edges in a graph by a few vertices and the distribution of contractible elements in graphs and matroids were also considered.

ACCOMPLISHMENTS (throughout award period): The simulation of the graph 3-coloring algorithm of the Antivoter model was very successful. In addition, we have proved a theoretical result on the convergence of the algorithm. For the longest circuit problem, we solved the 2-connected matroid case. We also solved the cographic cases for 2-, 3-, and 4-connected cases. We have completed several papers on this research (see attached list).

CONCLUSIONS: As a result of the project, we have produced several papers. One of the Co-PI's has written a book to be published by Prentice Hall.

Dr. Denley and Joshua Hanes, a graduate student, showed by simulation that the Antivoter model is very successful in solving the graph 3-coloring problem. Recently, they proved some

theoretical results about the anti-voter coloring algorithm as well. These results establish that the anti-voter coloring algorithm almost surely (with very high probability) successfully colors a 3 chromatic graph in two passes through the vertices, provided the graph is neither too sparse, nor too skewed. These results are being written up into a paper, to be submitted in the near future.

Dr. Reid and Nolan McMurray, an African-American doctoral student, are writing a paper on the longest cycle problem. They have proved the longest cycle conjecture for the case of maximum circuit size four. This problem is being reformulated in the dual case for cocircuits in matroids and bonds in graphs.

Tutte showed that the graph G has at least one contractible edge. Thomassen gave a simple proof of this fact and showed that contractible edges have many applications. In our paper, we show that there are at most $|V(G)|/5$ vertices that are not incident to contractible edges in a 3-connected graph G. This bound is best-possible. We also show that if a vertex v is not incident to any contractible edge in G, then v has at least four neighbors having degree three and each such neighbor is incident to exactly two contractible edges.

Dr. Wu and Joe Anderson, another African-American doctoral student, are working on an extremal problem on contractible edges. They characterize all 3-connected graphs with exactly $|V(G)|/5$ vertices that are not incident to contractible edges.

Let G be a simple 3-connected graph in the following. Ota and Saito proved that the contractible edges of G cannot be covered by two vertices. In another paper, Saito went on to prove that if the contractible edges are covered by exactly three vertices, then those three vertices must form a vertex cut of G. Hemminger and Yu used Saito's result to give a complete characterization of all such graphs. A paper of ours provides a short proof of both of these results.

An edge e of G is essential if neither the deletion G\de e nor the contraction G/e is both simple and 3-connected. Tutte's Wheels Theorem proves that the only simple 3-connected graphs with no non-essential edges are the wheels. In earlier work, as a corollary of a matroid result, the authors determined all simple 3-connected graphs with at most two non-essential edges. A paper of ours specifies all such graphs with exactly three non-essential edges. As a consequence, with an exception of 10 classes of graphs, all 3-connected graphs have at least four non-essential edges.

<u>SIGNIFICANCE</u>: Simulations show that the 3-coloring algorithm colors a graph with several thousand vertices in one pass almost perfectly. The graph coloring problem is NP-Hard as are all significant assignment and scheduling problems. Hence an understanding of why our algorithm works so well on this specific problem can lead to progress in randomly solving important problems in combinatorial optimization. The longest cycle problem is also related to important NP-Hard problems. Therefore progress on these problems has important scientific and commercial applications. Some of these problems are directly related to the assignment problem mentioned in Sailor 21. In addition, this grant has supported research that will contribute to at

least three doctoral dissertations with two of these dissertations being by African-American students.

PATENT INFORMATION: None

AWARD INFORMATION: James Reid named Managing Editor of the Proceedings of the Louisiana/Mississippi Section of the Mathematics Association of America.

REFEREED PUBLICATIONS (for total award period):

1. Denley, T. (2001) On a conjecture of Häggkvist on partial Latin Squares. Proceedings of the Thirty-Second Southeastern International Conference on Combinatorics, Graph Theory and Computing. 150: 73-78.

2. Denley, T. (2002) Arithmetic progressions of cycles in outer-planar graphs. Discrete Mathematics. 249: 65-70.

3. Denley, T. and Wu, H. (2001) A generalization of a theorem of Dirac. Journal of Combinatorial Theory Series B. 82: 322-326.

4. Reid, T.J. and Wu, H. (2001) On minimally 3-connected binary matroids. Combinatorics, Probability, and Computing. 10: 453-461.

5. Reid, T.J. and Wu, H. (2002) On elements in small cocircuits in minimally k-connected graphs and matroids. Discrete Mathematics. 243: 273-282.

6. Wu, H. Contractible elements in graphs and matroids, accepted by the journal Combinatorics, Probability, and Computing.

7. McMurray, N., Reid, T.J., Wei, B., and Wu, H. On intersection of largest circuits in connected matroids, submitted.

8. Oxley, J.G. and Wu, H. The 3-connected graphs with exactly three non-essential edges, submitted.

9. Denley, T. Coloring using the Antivoter model. In preparation.

10. McMurray, N. and Reid, T.J. Circuit intersections in matroids. In preparation.

11. Tan, F. and Wu, H. On covering contractible edges in 3-connected graphs. In preparation

12. Wu, H. On the distribution of vertically contractible elements in matroids. In preparation

BOOK CHAPTERS, SUBMISSIONS, ABSTRACTS AND OTHER PUBLICATIONS (for total award period)

1. Denley, T. Voyages in Mathematics--an undergraduate text to be published by Prentice Hall in spring 2003.